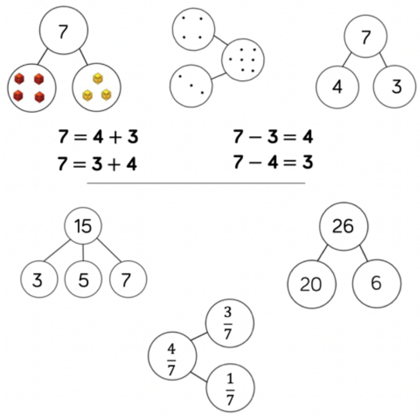
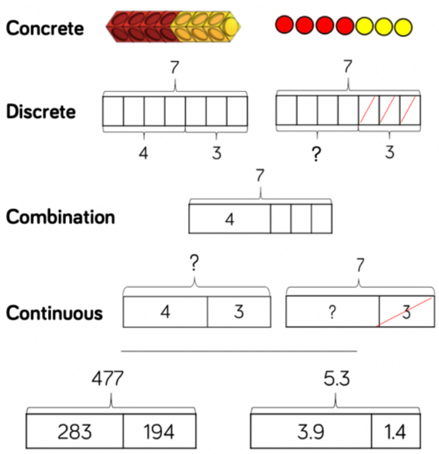
Western Primary School

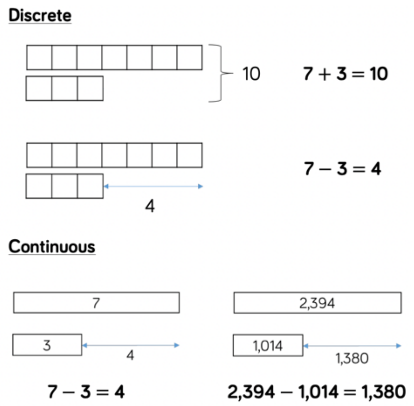
Maths Calculation Policy: Varied Manipulative Representations

Embedding the CPA (Concrete, Pictorial, Abstract) Approach

(based on the White Rose Maths Calculation Policy)

Addition and Subtraction

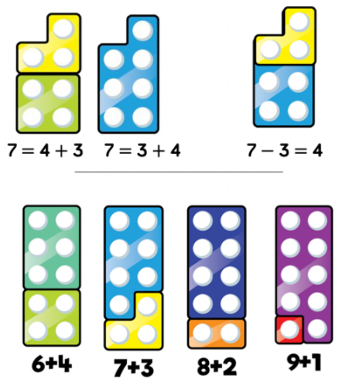
Representation	Benefit
	**Part-whole models** support children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model. Aggregation = combining numbers Partitioning = separating numbers into smaller part. In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.
	The **single bar model** is another type of a part-whole model that can support children in representing calculations to help them unpick the structure. Cubes and counters can be used in a line as a concrete representation of the bar model. In KS2, children can use bar models to represent larger numbers, decimals and fractions.



The **multiple bar model** is a good way to compare quantities whilst still unpicking the structure.

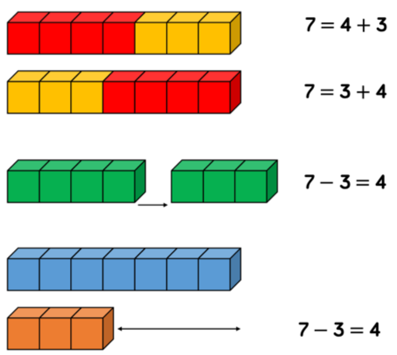
Two or more bars can be drawn, with a bracket labelling the whole positioned on the right-hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.



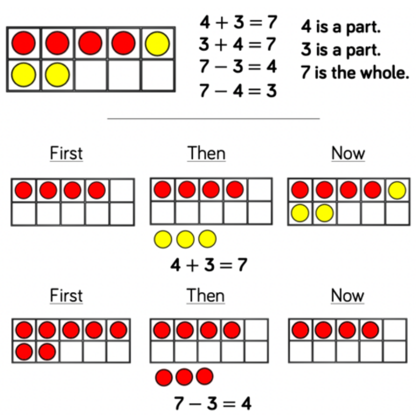
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing.



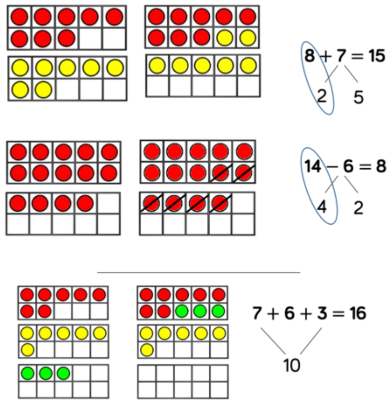
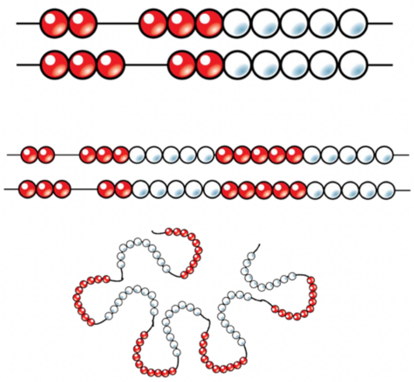
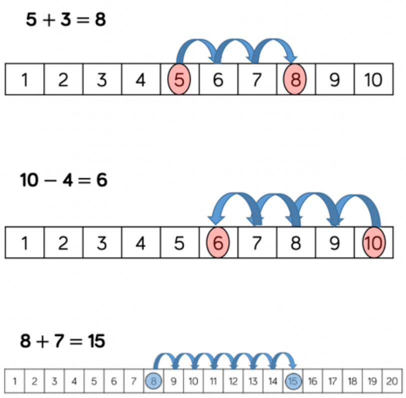
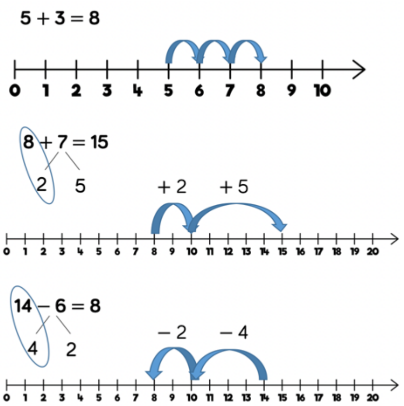
Cubes can be useful to support children with the addition and subtraction of one-digit numbers. Cubes can also be useful to look at subtraction as difference.

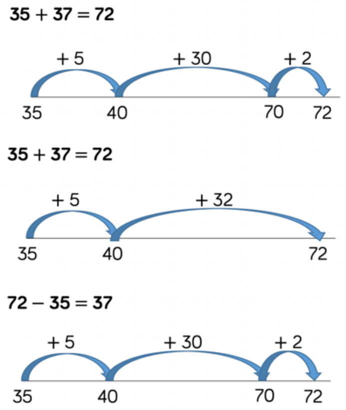
Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.



When adding and subtracting within 10, the **ten frame** can support children to understand the different structures of addition and subtraction.

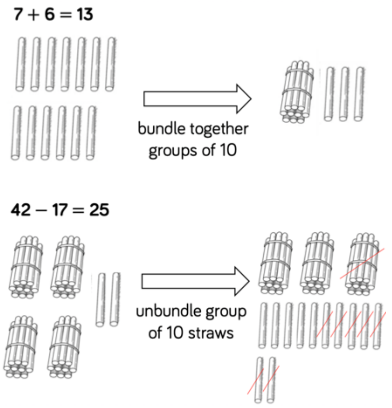
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

	When adding two single digits, children can make each number on separate **ten frames** before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.
	Different sizes of **bead strings** can support children at different stages of addition and subtraction. Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20. Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.
	**Number tracks** are useful to support children in their understanding of augmentation and reduction. Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.
	**Labelled number lines** also support children in their understanding of addition and subtraction as augmentation and reduction. These can be supported by ten frames.



Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

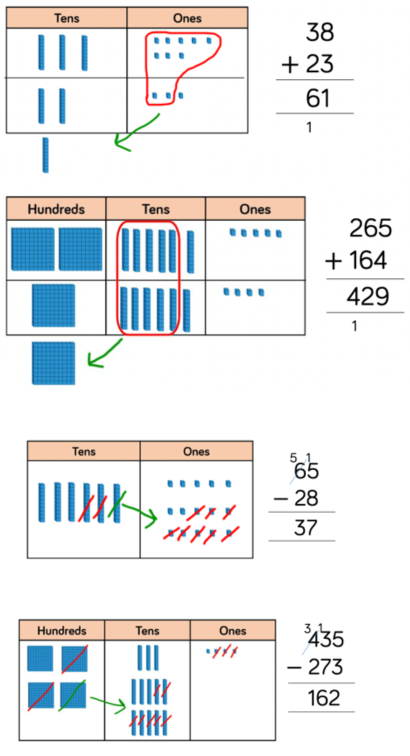
Developing from labelled number lines, children can add or subtract by jumping to the nearest 10 and then adding or subtracting the rest of the number either as a whole or by adding or subtracting the tens and ones separately.



Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

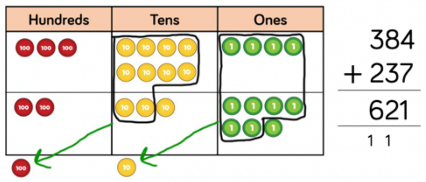
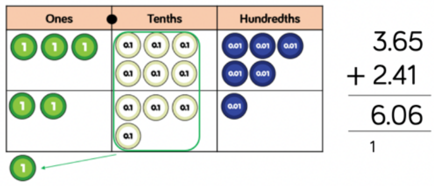
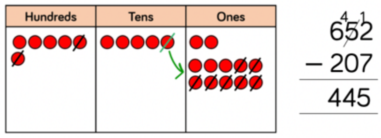
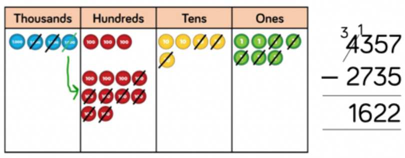
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

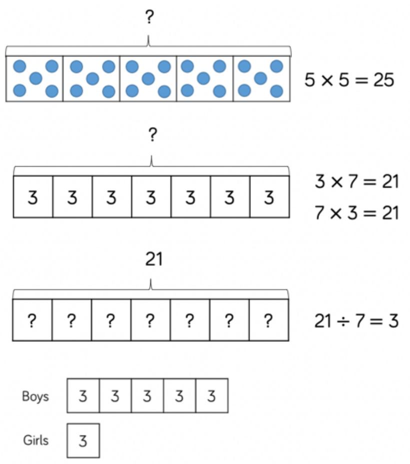
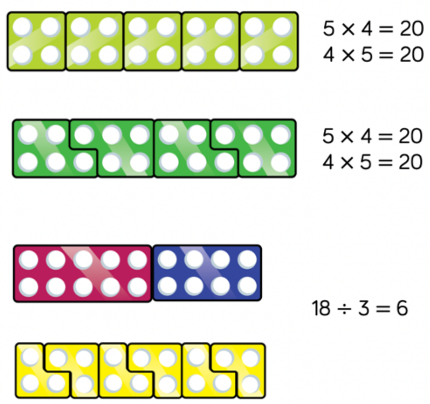
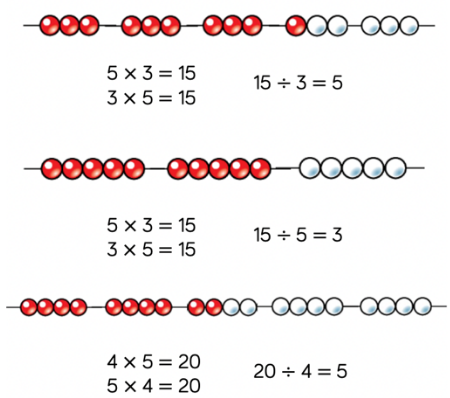


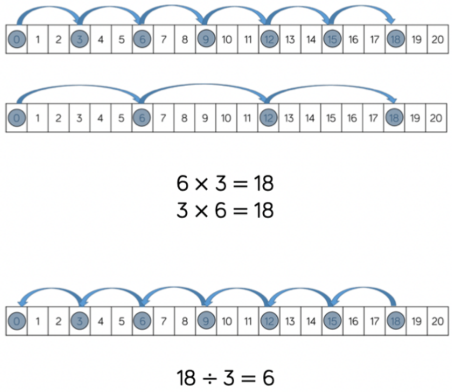
Using **Base 10** or **Dienes** is an effective way to support children's understanding of column addition and subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

   	Using **place value counters** is an effective way to support children's understanding of column addition and subtraction. Much like the use of Base 10, children should first add and subtract without an exchange before moving on to addition and subtraction with exchange.

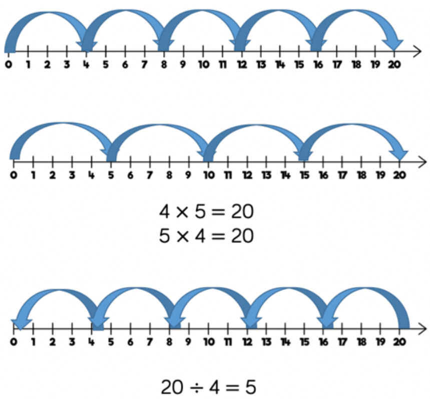
Multiplication and Division

Representation	Benefit
	Children can use the **single bar model** to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication. Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups. It is important when solving word problems that the bar model represents the problem.
	**Number shapes** support children's understanding of multiplication as repeated addition. Children can build multiplications in a row using the number shapes. When dividing, number shapes support children's understanding of division as grouping.
	**Bead strings** to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count. When dividing, children build the number they are dividing and then group the beads into the number they are dividing by.



Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

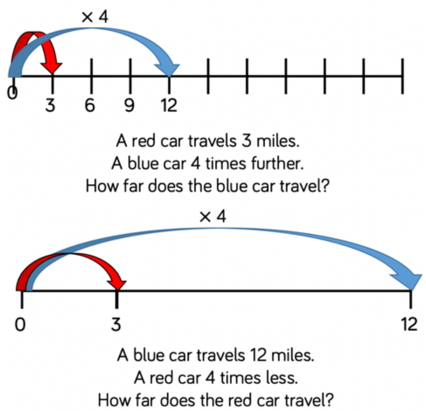


Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

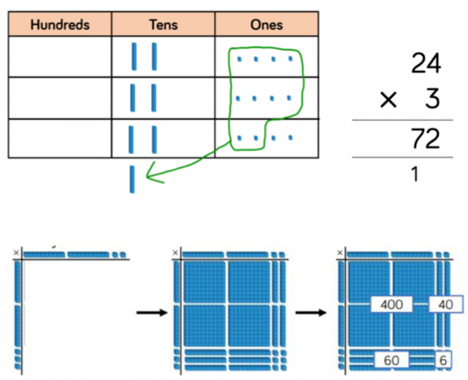
Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.



Children can use **blank number lines** to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.



Using **Base 10** or **Dienes** is an effective way to support children's understanding of multiplication and division.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

When numbers become larger in division, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base

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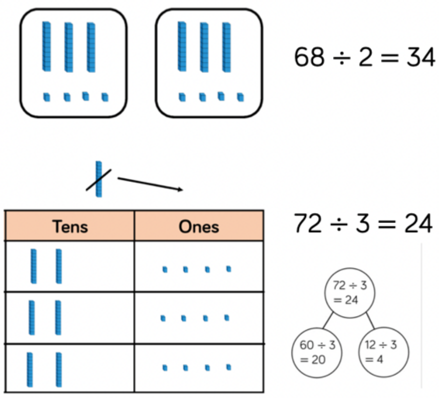
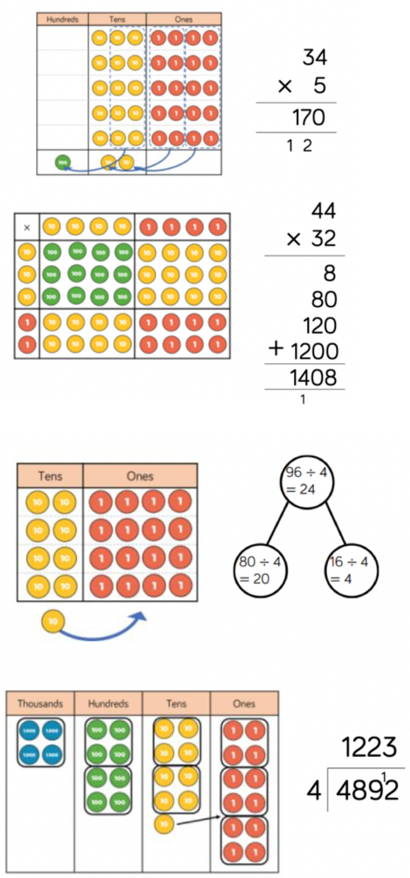
<td>
</td>
<td>10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.</td>
</tr>
<tr>
<td>
</td>
<td>Using place value counters is an effective way to support children's understanding of column multiplication and division. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

When working with smaller numbers, children can use place value counters to share between groups. This method can be linked to the part-whole model to support children to show their thinking. Place value counters also support children's understanding of short division by grouping the counters rather than sharing them.</td>
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